Entanglement: Spooky Action at a Distance

Julien Ross

August 2025

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The Paradox: Sometimes, measuring one qubit instantly changes the state of the another qubit. It doesn't matter how far apart the qubits are from one another.

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The Paradox: Sometimes, measuring one qubit instantly changes the state of the another qubit. It doesn’t matter how far apart the qubits are from one another.

Huh?!

Can this really happen instantaneously? Einstein sarcastically called this *spooky action at a distance*.

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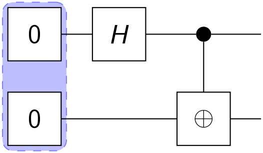
Experiments would later show that what seemed to be a paradox was just another counterintuitive property of the world we live in.

We now call this phenomenon *quantum entanglement*. That sounds far less spooky!

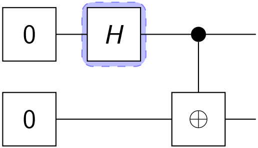
We will now create a *Bell pair* – the simplest example of entanglement.

Preparing a Bell Pair



Initialize both qubits to $|0\rangle$.

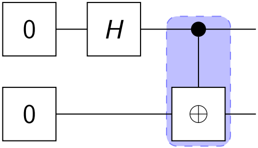
Preparing a Bell Pair



Hadamard Gate: Changes top $|0\rangle$ to $|+\rangle$.

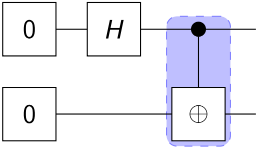
- **Before**: 100% chance of $|00\rangle$.
- **After**: 50% chance of $|00\rangle$, 50% chance of $|10\rangle$.

Preparing a Bell Pair



Controlled-NOT: Flips bottom qubit if top qubit is $|1\rangle$.

Preparing a Bell Pair



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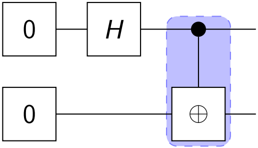

Controlled-NOT: Flips bottom qubit if top qubit is $|1\rangle$.

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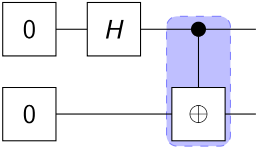

Controlled-NOT: Flips bottom qubit if top qubit is $|1\rangle$.

- **Before**: 50% chance of $|00\rangle$, 50% chance of $|10\rangle$.
- **After**: 50% chance of $|00\rangle$, 50% chance of $|11\rangle$.

Measuring a Bell Pair

There are two possible measurement outcomes:

- $|00\rangle$ (50% chance)
- $|11\rangle$ (50% chance)

When we measure one qubit, we learn the state of the other qubit without checking!

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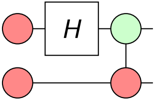
The Bell Pair

This particular configuration of probabilities and outcomes, involving both qubits, is what we call a *Bell Pair*

Exercise: Simplify the Diagram

$\Phi^+ =$  $=$

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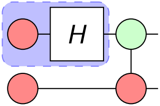
$\Phi^+ =$  $=$

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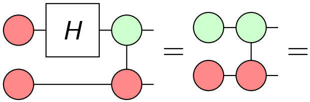
$\Phi^+ =$ 

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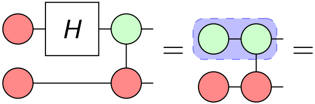
$\Phi^+ =$ 

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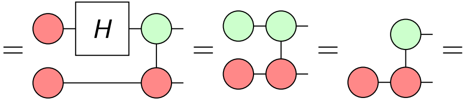
$\Phi^+ =$ 

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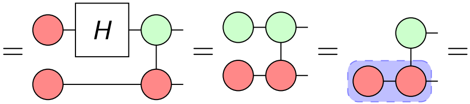
$\Phi^+ =$ 

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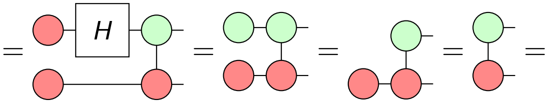
$\Phi^+ =$ 

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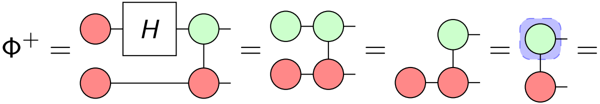

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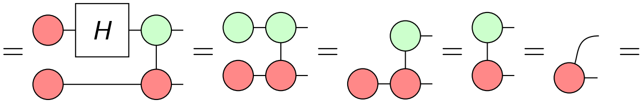

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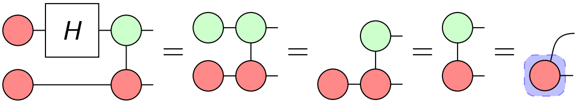
$\Phi^+ =$  $=$

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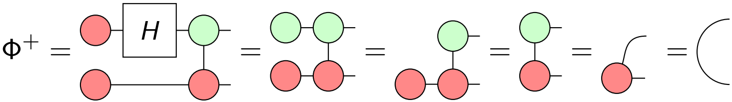
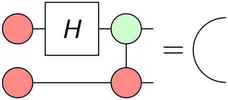
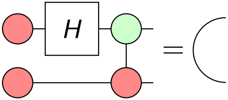


Bell Pair in ZX-Calculus



Our circuit is a bent wire–it connects two qubits in the sense that measuring one influences the other.

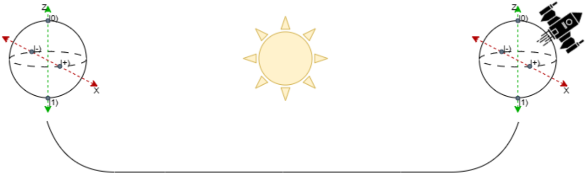
Bell Pair in ZX-Calculus



Our circuit is a bent wire–it connects two qubits in the sense that measuring one influences the other.

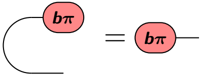
This is *quantum entanglement*!

Entanglement *Does Not* Depend on Distance



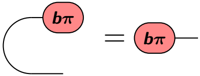
No matter how far apart the entangled qubits are, measuring one still determines the state of the other

Entanglement and Measurement



We can freely (un)bend wires in the ZX-calculus.

Entanglement and Measurement



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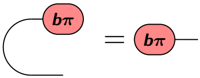
Question Time!

Why does this equation tell us about Bell pairs?

Entanglement and Measurement



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Question Time!

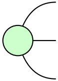
Why does this equation tell us about Bell pairs?

The Answer

A measurement outcome from the first qubit becomes the state of the second qubit.

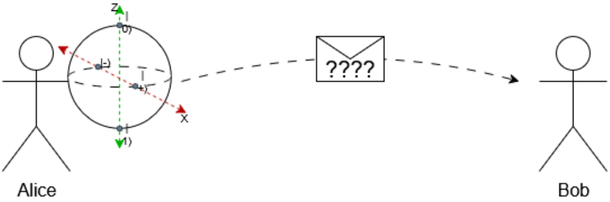
There Are Many Ways to Entangle Qubits!

One of tomorrow's challenges introduces another circuit that achieves entanglement—this time, for 3 qubits!

$$|GHZ\rangle =$$ 

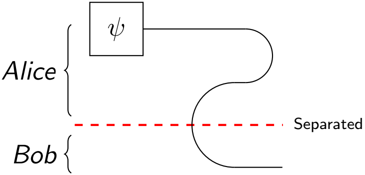
Quantum Teleportation: Scenario

Imagine that there are two friends named Alice and Bob. Alice has a qubit in an arbitrary state $|\psi\rangle$.



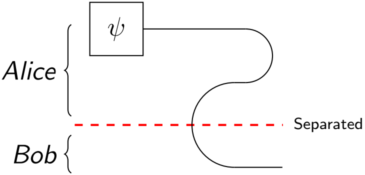
Alice wants to send her qubit to Bob. She could carry it to him, but there's a risk it could get stolen along the way. Instead, she decides to *teleport the data*!

Quantum Teleportation: The Protocol



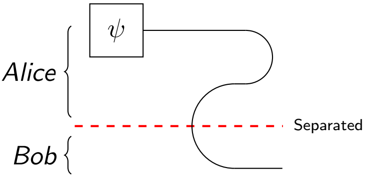
Alice and Bob are physically separated and Alice has a qubit in state $|\psi\rangle$.

Quantum Teleportation: The Protocol



This circuit has 1 input and 1 output...

Quantum Teleportation: The Protocol



...but we have 3 qubits mid-way through the circuit.

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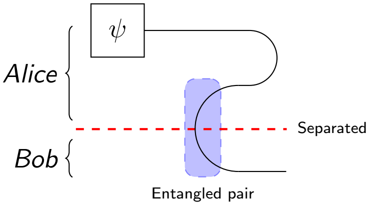
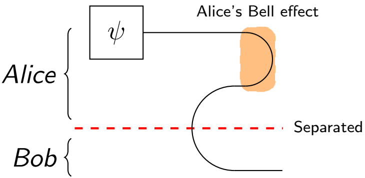


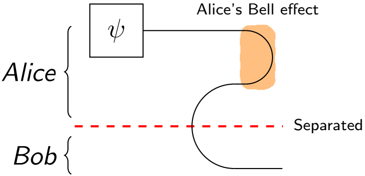
Alice and Bob share a Bell pair.

Quantum Teleportation: The Protocol



Alice performs a Bell effect on her $|\psi\rangle$ qubit and her entangled qubit from the pair.

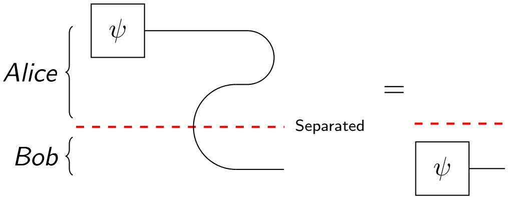
Quantum Teleportation: The Protocol



Alice performs a Bell effect on her $|\psi\rangle$ qubit and her entangled qubit from the pair.

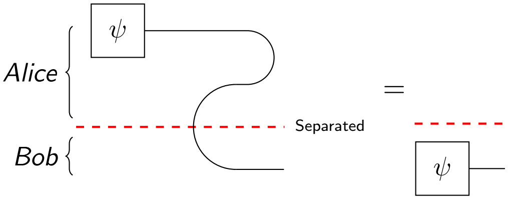
You can learn what this means in terms of circuits tomorrow. For now, it's a wire bent to the right.

Wiring Bending is Quantum Teleportation



If we pull $|\psi\rangle$ along the wire, then $|\psi\rangle$ ends up with Bob, as desired.

Wiring Bending is Quantum Teleportation



If we pull $|\psi\rangle$ along the wire, then $|\psi\rangle$ ends up with Bob, as desired.

Note that only the *state of the qubit* has been teleported, Alice still has the particle.