Opening the Box: Quantum Measurement

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August 2025

DALHOUSIE UNIVERSITY

Measurement and Collapse

What Is Measurement?

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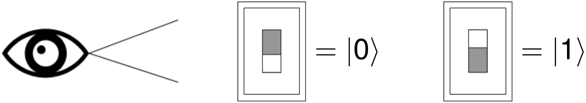

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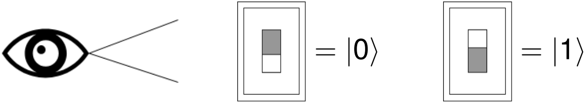

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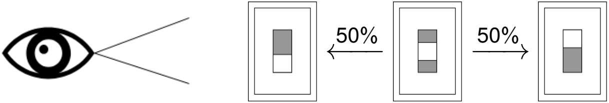

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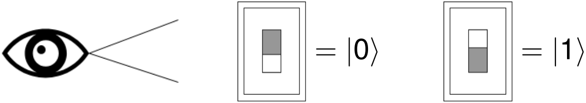

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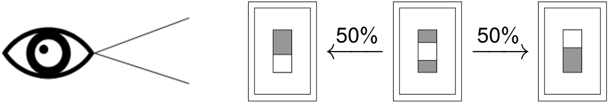

Things get even weirder with a qubit on the Bloch sphere! To see why, let us go back to the double slit experiment.

Double Slit Experiment Revisited

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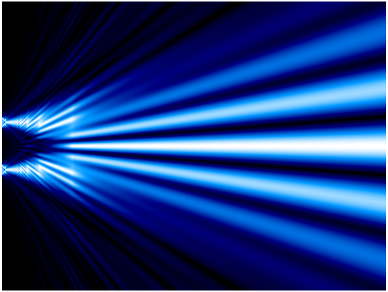

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In the rest of this lesson, we will learn why measurement is important, and how we predict measurement outcomes.

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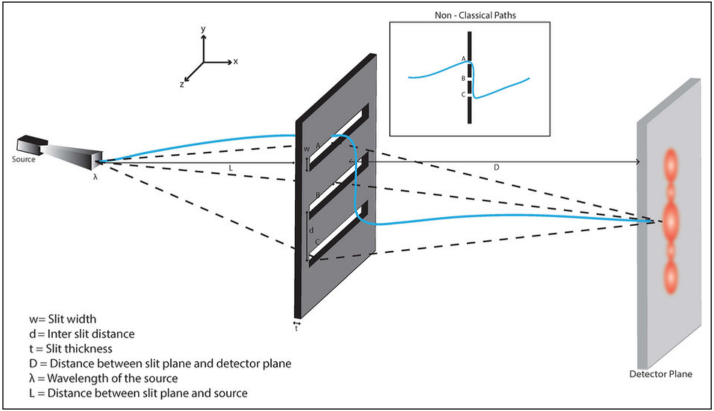

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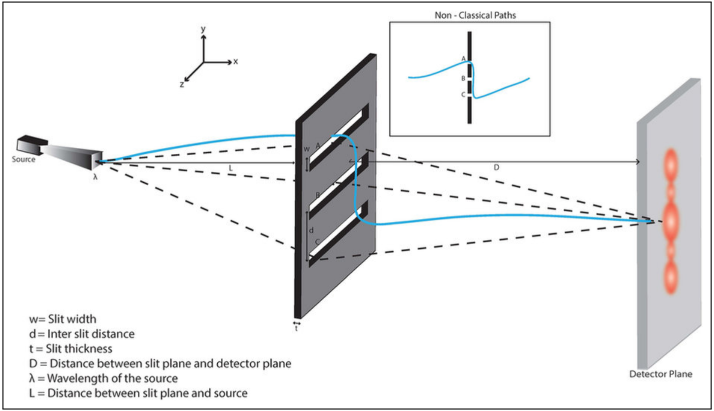

What would happen if we put a detector near one of the three slits?

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Heisenberg's uncertainty principle tells us *how* uncertain we are about these two things.

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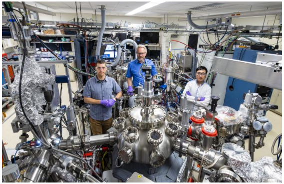
Then why do we perceive only one state of the superposition?

Measurement in Quantum Computing

Why Do We Measure?

In physics, measurement tells us about the states of particles.



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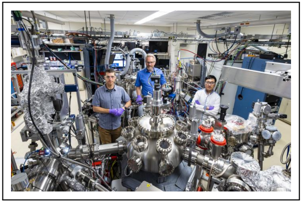

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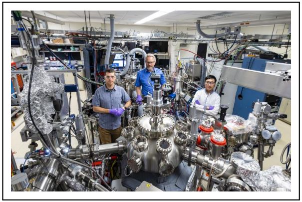

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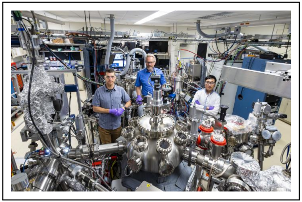

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- Using superposition, we can store and edit lots of data at once!

- Measurement is how we retrieve information from the superposition.

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- If the scan is of a real tumor, ideally we'd find a **high likelihood** of cancer, such as 95%.

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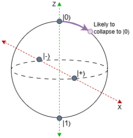

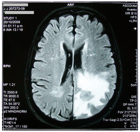

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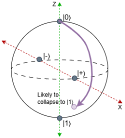

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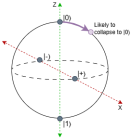
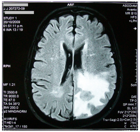
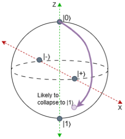
 $\mapsto$ 

 $\mapsto$ 

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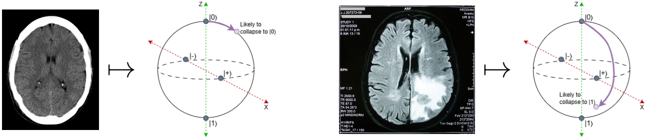

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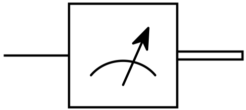
By running the algorithm many times, likelihood can be estimated.

Measurement in Quantum Computing

We add a notation of a new gate to our circuits for quantum measurement.

measure `result` $\mapsto$ 

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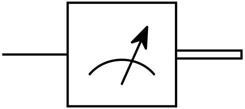
measure `result` $\mapsto$ 

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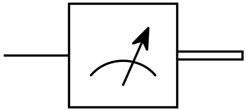
measure `result` $\mapsto$ 

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The measurement gate collapses the state of a qubit, and then spits out the measurement outcome as a conventional bit.

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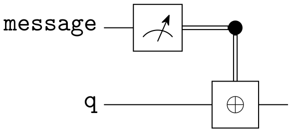
let `value` = **measure** `message`
if `value` **then negate** `q` $\mapsto$



Black dot: If we measure 1, we do the NOT gate on qubit q. Otherwise, we do nothing.

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Solution

We introduce a variable to capture the measurement outcome.

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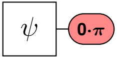
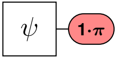
$\alpha =$ 

$\beta =$ 

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Born's Rule

What do we mean by *close*? Born's Rule tells us that the probability of measuring $|0\rangle$ will be $|\alpha|^2$ and the probability of measuring $|1\rangle$ will be $|\beta|^2$.

ZX-Diagrams and Measurement Rules

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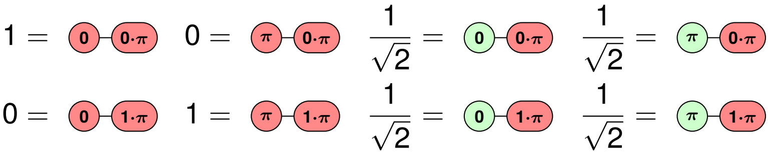

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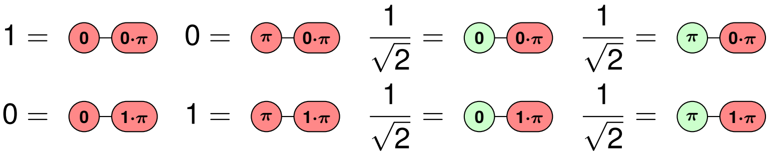

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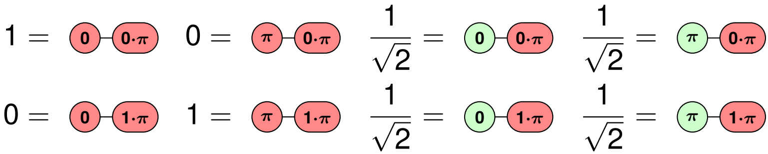

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What About Other Outcomes?

Computing these probabilities requires some tedious math and imaginary numbers! We will learn to use software to do this for us.

A Random Number Generator

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1. First, we will start with a qubit in state $|0\rangle$.
2. Then, we will apply a Hadamard gate to obtain a qubit in state $|+\rangle$.
3. Finally, we will measure the qubit to obtain a bit that is in state $|0\rangle$ half of the time, and in state $|1\rangle$ the other half.

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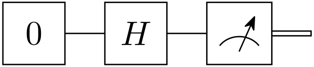
This program corresponds to the following circuit.



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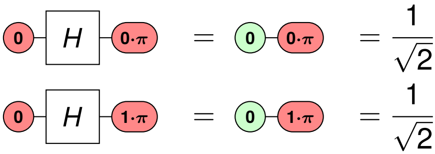

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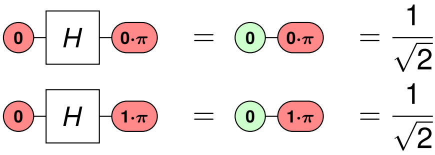

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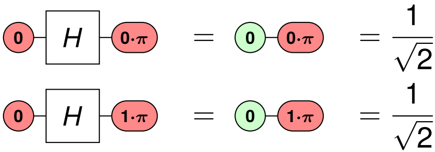

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Not Just an Idea

We would like to emphasize that this is not just a cool theoretical idea. Indeed, people have already built random number generators based on this simple idea!

Improved ZX Diagrams

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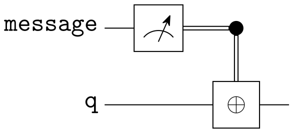
Let us return to the example of NOT gate controlled by measurement:



Outcomes as Variables

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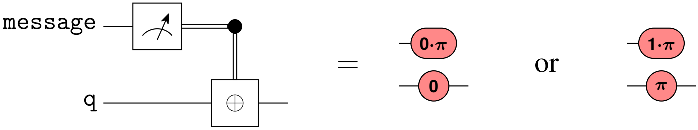

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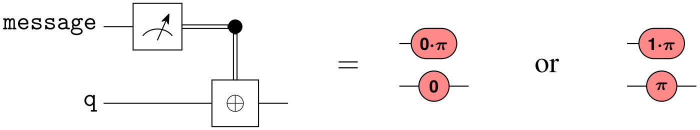

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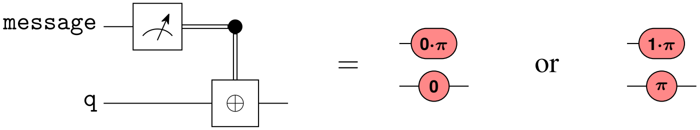

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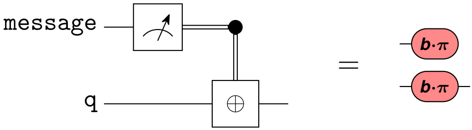
- If the measurement outcome is 1, then it negates q.

The second qubit q is rotated by $b \cdot \pi$ for each outcome b.



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